



# Part IV: Line Drawings and Perception

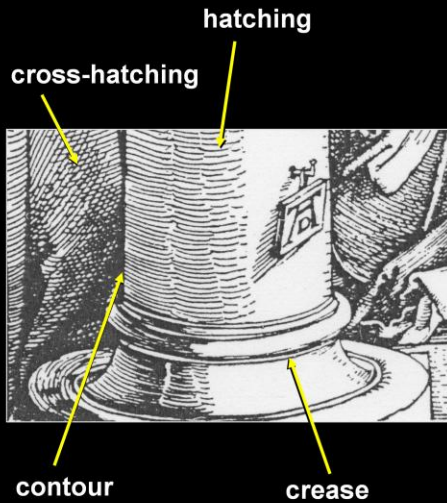
Doug DeCarlo

Line Drawings from 3D Models  
SIGGRAPH 2008

You've heard about the different types of lines that can appear in line drawings.

Now we're ready to talk about how people perceive line drawings.

# Line drawings



[Dürer 1505]

Line drawings bring together an abundance of lines to yield a depiction of a scene.

Take a look at this print by Dürer.

It uses different types of lines that convey geometry and shading in a way that's compatible with our visual perception.

We seem to interpret this scene easily and accurately.

\*\*\*

Some of the lines in this drawing only convey geometry.

But the fullness of this drawing comes from Dürer's use of hatching and cross-hatching.

These patterns of lines convey shading through their local density and convey geometry through their direction.

## Line drawings



[Flaxman 1805]

Other drawings rely on little or no shading.

In this drawing by Flaxman, shading is limited to the cast shadows on the floor.

The detail in the cloth here is conveyed with lines such as contours, creases, and maybe other lines such as suggestive contours, or ridges and valleys.

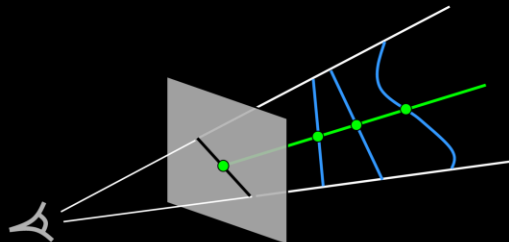
While artists can make drawings like this, they can't really explain what they're doing.

They rely on their training, and use their own perception to judge the effects of their decisions.

# Ambiguity of lines in images

## The ambiguity of projection

- an infinity of 3D curves project to the same line in the image



It's actually a little surprising that line drawings are effective at all.

At first, line drawings just seem to be too ambiguous.

An infinite number of 3D curves can project to the same line in the image.

All images have this ambiguity, but in photographs, there are many other visual cues, such as shading and texture, that help to indicate shape.

Here, we're just looking at individual lines.

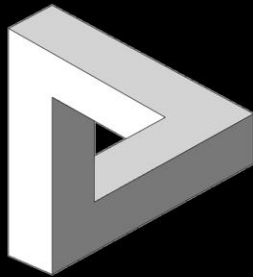
But it turns out that individual lines contain a wealth of information about shape.

This information is typically local in nature.

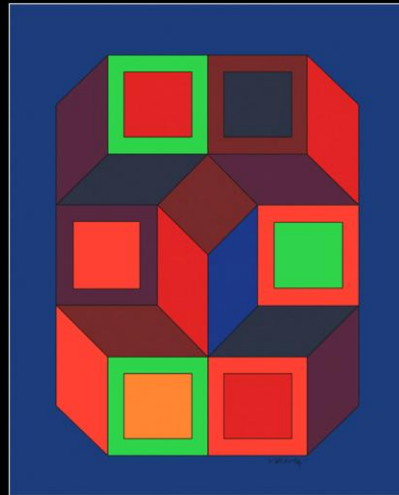
But our perception is somehow able to bring all of this together into a coherent whole.

Well, sort of.

# Impossible line drawings



The Penrose triangle [1958]



[Vasarely 1973]

Line drawings of impossible 3D objects show us that this coherence is NOT global.

The Penrose triangle, which was inspired by the work of Escher, is perhaps the simplest of the impossible figures.

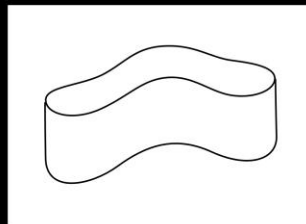
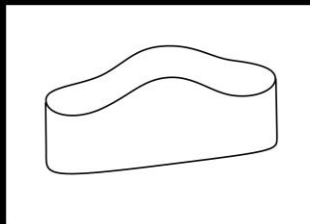
When you first look at it, it seems to be an ordinary object.

Closer inspection is a little unsettling, and its inconsistencies are easily revealed.

Vasarely pushed this idea even further, and made pictures such as this one that encourage us to explore several different inconsistent interpretations at the same time.

# Interactions between lines

- Line drawing interpretation depends on non-local context.



after [Barrow 1981]

Although you might think the Penrose triangle shows that there are no global effects for visual inference, it's not that easy.

Take a look at these two drawings.

The figure on the left appears to be raised in the center, while the figure on the right appears to have a flat top, and bends along its length.

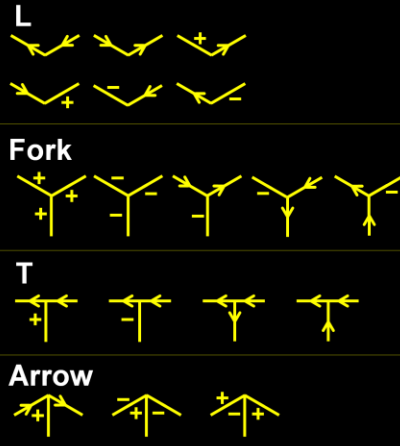
If we compare these two drawings line by line, the only difference is the line along the bottom.

Nobody knows whether we see this difference because we consistently integrate local information, or perform certain types of non-local inference.

# Interpretation of line drawings

## Labeling polyhedral scenes

- catalog of junction labels
- find consistent labelings using constraint satisfaction [Waltz 1975]



adapted from [Waltz 1975]

Use of non-local inference is plausible.

Algorithms exist for searching among the space of possibilities.

Waltz's method for line-labeling starts with catalogs of all possible line junctions, which are places where two or more lines meet.

Here's the catalog of 18 junctions that lets you classify any trihedral vertex in a polyhedral scene.

CONVEX lines are labeled with a PLUS, CONCAVE lines with a MINUS.

Arrows mark visual occlusions, where the closer surface is to the RIGHT of the arrow.

Algorithms for constraint satisfaction compute all possible configurations of junctions for a particular picture.

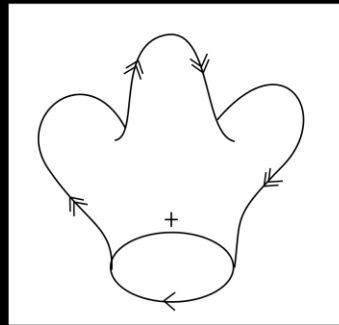
For an impossible figure, this set is empty.

# Interpretation of line drawings

## Labeling for more general scenes

- larger catalogs of junctions
- prune unreasonable or unlikely configurations

[Barrow 1981, Malik 1987]



after [Malik 1987]

This idea can be extended for line drawings that contain smooth surfaces.

First, you need a more comprehensive junction catalog.

Then, you need methods that can prune away large numbers of unreasonable interpretations, to prevent a combinatorial explosion.

These algorithms only label lines with a type.

They don't infer geometry.

Furthermore, existing algorithms are restricted to lines from contours and creases, and sometimes lines from shadows.



# Interpretation of line drawings

Unfortunately, we do not know how humans process line drawings.

Nevertheless, much is known about the information humans might be using.

While these algorithms suggest that exhaustive search might be a viable method for scene interpretation, they don't say anything directly about how PEOPLE interpret line drawings.

In fact, not very much is known about that.

Even so, we can still be very specific about what INFORMATION is available in a line drawing.

This is the information that our perceptual systems are probably using.

# Interpretation of line drawings

Each line **constrains** the depicted shape

- depending on the type of line

The type can sometimes be inferred from context (within the drawing)

Ambiguity always remains

- some interpretations are more likely than others

Essentially, each line in a drawing places a constraint on the depicted shape.

In the end, the geometry that results is never unique.

But our perceptual systems excel at uncovering the most reasonable and most likely interpretations.

So now let's go through the kinds of information that different types of lines provide.

# Information in line drawings

## Lines can mark **fixed** locations on the shape

- creases (sharp folds)
- ridges and valleys
- surface markings (texture features, material boundaries, ...)
- hatching lines (although density is lighting-dependent)

## Lines can mark **view-dependent** locations on the shape

- contours (external and internal silhouettes)
- suggestive contours
- apparent ridges

## Lines can mark **lighting-dependent** locations on the shape

- isophotes (boundaries of attached shadows or in cartoon shading)
- edges (i.e. boundaries of cast shadows)

First, we'll consider lines that mark **FIXED** locations on a shape, such as creases, ridges and valleys, and surface markings.

Then, we'll consider **VIEW DEPENDENT** lines. The most important is the **CONTOUR**, which lets us infer surprisingly rich information about the shape.

There are also lines whose locations are lighting-dependent, such as edges of shadows, but I'm not going to be discussing those.

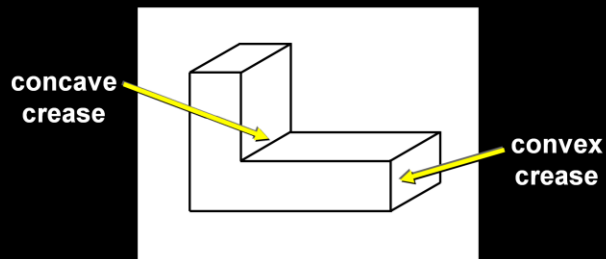
Of all these lines, creases and contours are well understood.

Research on the information other types of lines provide is ongoing.

# Information in creases

Creases mark discontinuities in surface orientation  
– often a luminance discontinuity in shaded imagery

Convex vs. concave: cannot be determined locally



Creases mark discontinuities in surface orientation,  
and are typically visible in a REAL image  
as a discontinuity in tone.

The crease can be concave or convex.

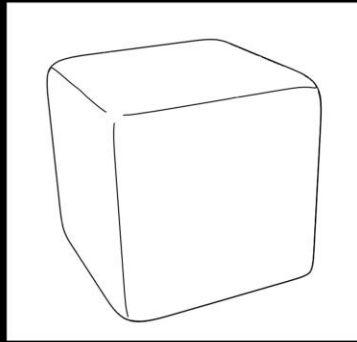
But local information doesn't let us determine which.

The algorithms for line labeling I mentioned earlier  
proceeded by considering every possibility,  
and then enforced consistency across the whole drawing.

# Information in ridges and valleys

Mark locally rapid changes in surface orientation

- one possible extension of creases to smooth surfaces
- like creases, locally indistinguishable



Ridges and valleys mark locally maximal changes in surface orientation.

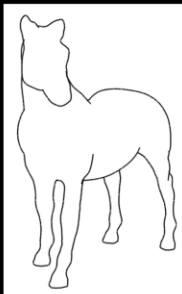
In real images, they can appear as smooth but sudden changes in tone.

The ridges on this rounded cube are particularly effective at conveying its shape, when drawn with the contours.

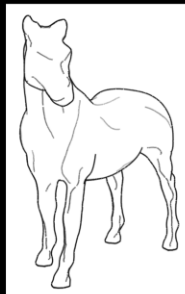
# Information in ridges and valleys

## Their use is still unresolved

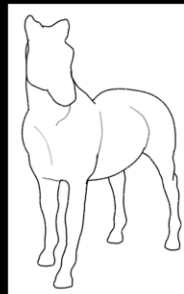
- many seem to convey shape
- others seem to convey surface markings
- humans can locate them in shaded imagery [Phillips 2003]



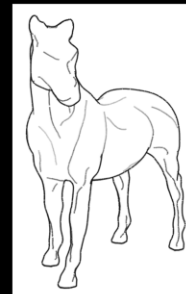
contours



with ridges



with valleys



with both

Research on the use of ridges and valleys in line drawings is ongoing.

When used alongside contours, ridges and valleys can produce an effective rendering of a shape.

The valleys on the side of the horse are quite convincing.

In other cases, they look like surface markings, such as the ridges on its head.

Ridges and valleys are reasonable candidates for line drawings, as there is psychological evidence that viewers can reliably locate them in realistic images.

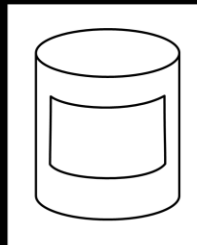
# Information in surface markings

Convey shape when they lie along *geodesics*  
(locally shortest paths on the surface)

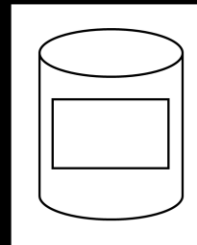
[Stevens 1981, Knill 1992]

Related to perception of texture

[Knill 2001]



along geodesics  
of cylinder



not along geodesics  
of cylinder

Markings on a surface can appear as arbitrary lines inside the shape.

However, for a certain type of line known as a geodesic, they can also convey shape.

Geodesics are simply lines on the surface that are locally shortest paths.

Stevens points out that for many fabricated objects, surface markings are commonly along geodesics.

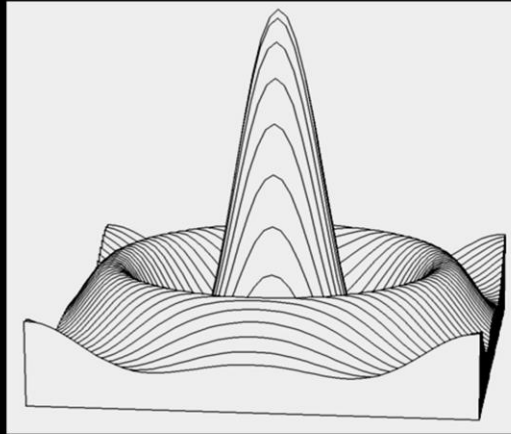
Take for instance the label on the cylinder on the left.

For a more general class of surfaces, Knill draws connections between texture patterns and sets of parallel geodesics.

# Information in surface markings

Parallel lines in space can also convey shape

[Stevens 1981]



When used in repeating patterns,  
other curves can be effective as well.

Sets of parallel lines,  
which are often used to construct plots of 3D functions,  
are one notable example.

The images that result are analogous to using  
a periodic solid texture.

Stevens points out that all one needs to do  
to infer the shape is  
to build correspondences between adjacent lines,  
matching up points with equal tangent vectors.



# Information in hatching

## Conveys shape through direction of hatching lines

- more effective when drawn along geodesics

[Stevens 1981, Knill 2001]

- lines of curvature are particularly effective

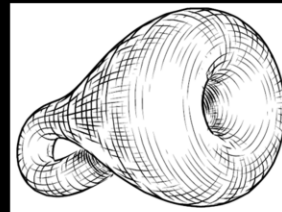
[Girshick 2000, Hertzmann 2000]



along geodesics



along lines of curvature



[Hertzmann 2000]

The use of repeating patterns of lines forms the basis of hatching.

These lines convey shape in two different ways; they convey shape directly when they are drawn along geodesics.

And they convey shape indirectly through careful control of their density, which can be used to produce a gradation of tone across the surface.

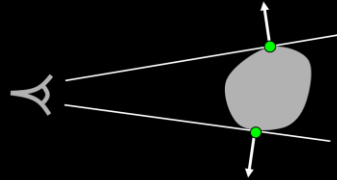
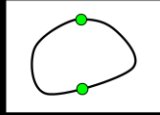
Particularly effective renderings are obtained when lines of curvatures are used, which align with the principal directions of the surface. These also happen to be geodesics.

So that's it for lines whose locations are **FIXED** on the shape.

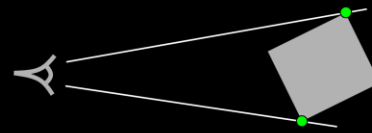
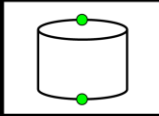
# Information in contours

Contours emerge in two situations:

1. Smooth parts of the shape where  $n \cdot v = 0$



2. Creases separating front- and back-faces



Next are lines whose location depends on the viewpoint.

The contour is the most notable example.

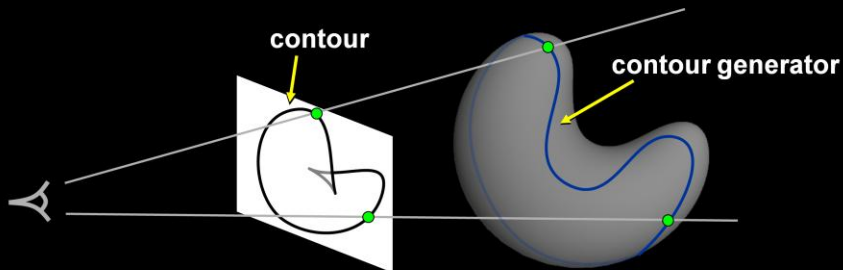
There are two situations when contours are formed.

On a smooth surface, contours are produced when the surface is viewed edge-on.

On an arbitrary surface, contours can also appear along a crease.

# Information in contours

The *contour generator* is the curve sitting on the surface that projects to the occluding contour



In either case,  
sitting on the surface  
is a 3D curve known as the **CONTOUR GENERATOR**.

This curve marks all local changes in visibility across the shape.

For a typical viewpoint, the contour generator  
consists of a set of isolated loops.

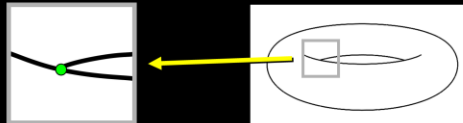
It projects into the image to become the contour.

So not all parts of the contour are visible.

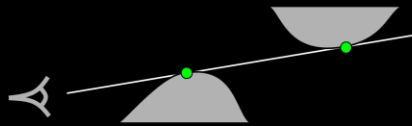
# Visibility of contours

Not all parts of the contour generator are visible

- First case: “non-local” occlusions
  - T-junctions



– viewing direction grazes surface at two locations



Let's consider the different cases of visibility for contours.

On a smooth surface,  
the first case is when one part of the shape  
occludes another more distant part.

This appears in the image as a T-junction,  
where the contour goes behind another part of the shape.

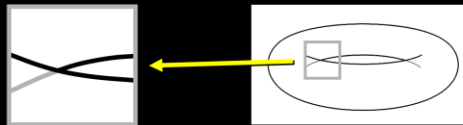
\*\*\*\*

At the location where the visibility changes,  
the visual ray is tangent to the surface in two places.

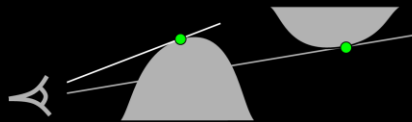
# Visibility of contours

Not all parts of the contour generator are visible

- First case: “non-local” occlusions
  - The contour continues in back



- and is simply occluded



The contour then continues behind the shape,

\*\*\*\*

and is occluded.

This can be seen in this transparent line-drawing of a torus.

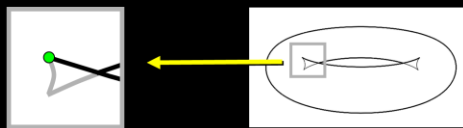
# Visibility of contours

Not all parts of the contour generator are visible

- Second case: ending contours
  - the visible part of the contour ends



– at a cusp in the projection of the contour generator



The second case occurs  
where the contour comes to an end in the image.

An ending contour.

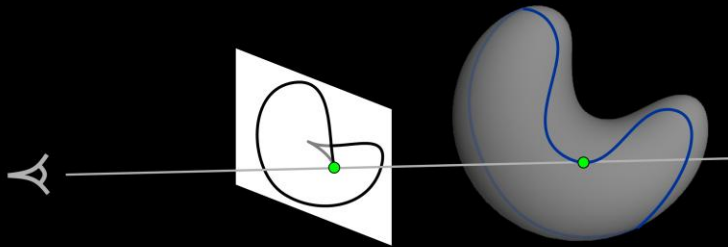
\*\*\*\*

When the occluded part of the contour continues,  
it does so at a cusp in the contour.

# Visibility of contours

Not all parts of the contour generator are visible

- Second case: ending contours
  - the tangent of the contour generator lines up with the viewing direction

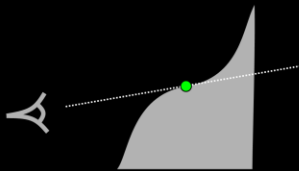


This cusp occurs because the contour generator lines up with the viewing direction, so that its tangent projects to a point.

# Visibility of contours

Not all parts of the contour generator are visible

- Second case: ending contours
  - the radial curvature is zero



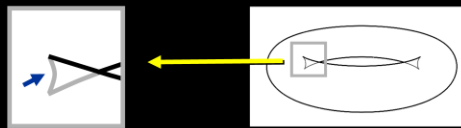
At an ending contour, the radial curvature is zero, which means that we're looking along an inflection; an asymptotic direction of the surface.



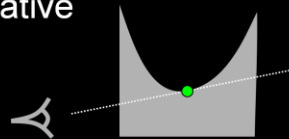
# Visibility of contours

Not all parts of the contour generator are visible

- Third case: local occlusions (“inside contours”)
  - these parts of the contour generator are never visible; the surface always blocks them



– the radial curvature is negative



The last case is a local occlusion;  
places where the surface has no choice but to occlude itself.

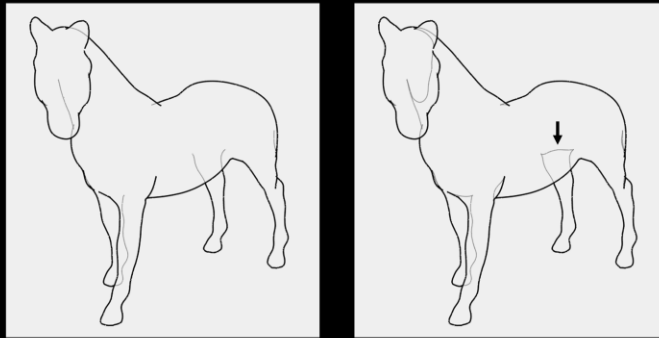
\*\*\*\*

These are locations where the radial curvature is negative.

# Visibility of contours

Not all parts of the contour generator are visible

- Third case: local occlusions (“inside contours”)
  - can be confusing in transparent renderings



In transparent renderings of contours, one typically does not draw the local occlusions, as the results can be confusing.

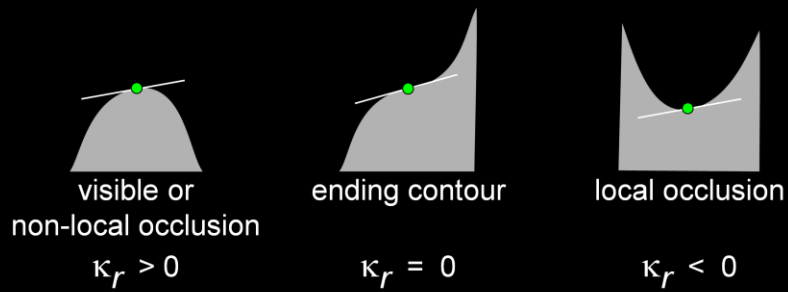
The image on the right here draws these contours.

One is marked with an arrow.

These curves actually correspond to regular contours for an inside-out version of the surface.

# Visibility of contours

The three cases (on smooth surfaces):



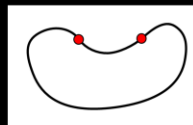
Here are the three cases, all together.

# Apparent curvature of contours

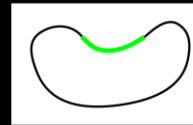
The apparent curvature  $\kappa_{app}$  is the curvature of the contour in the drawing (or image)



$$\kappa_{app} > 0$$



$$\kappa_{app} = 0$$



$$\kappa_{app} < 0$$

At the cusp of an ending contour,  $\kappa_{app}$  is infinite

Now, let's consider what the contours look like in the image.

The apparent curvature is simply the curvature of the contour in the drawing.

The convex parts of the contour have positive apparent curvature,

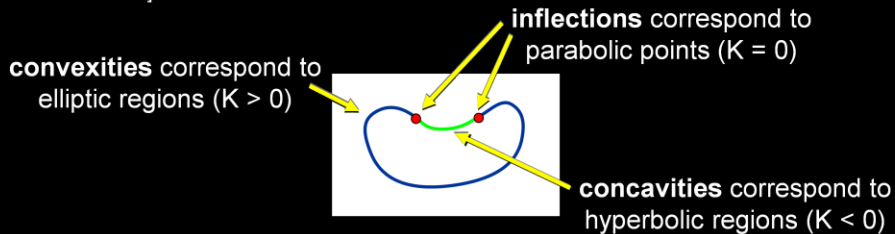
the concave parts have negative apparent curvature, and it's zero at the inflections.

At the ending contours, the apparent curvature is infinite due to the cusp.

# Apparent curvature of contours

At a point on the contour (of a smooth surface):

- sign of apparent curvature  $\kappa_{app}$  = sign of Gaussian curvature  $K$   
[Koenderink 1984]



- Koenderink proves  $K = \kappa_r \frac{\kappa_{app}}{d}$ 
  - $d$  is the distance to the camera
  - $\kappa_r \geq 0$  for visible points on the contour

Koenderink proved a surprising and important relationship between the apparent curvature and the Gaussian curvature.

Specifically, for visible parts of the contour on a smooth surface, they have the same sign.

This means we can infer the sign of the Gaussian curvature simply by looking at the contour.

\*\*\*\*

**CONVEX** parts of the contour correspond to locations where the Gaussian curvature is **POSITIVE**: elliptic regions.

\*\*\*\*

**INFLECTIONS** on the contour correspond to locations where the Gaussian curvature is **ZERO**.

\*\*\*\*

**CONCAVE** parts of the contour correspond to locations where the Gaussian curvature is **NEGATIVE**: saddle-shaped regions.

\*\*\*\*

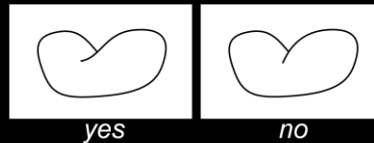
Koenderink gives a formula that connects these two quantities, that involves the distance to the camera and the radial curvature.

# Apparent curvature of contours

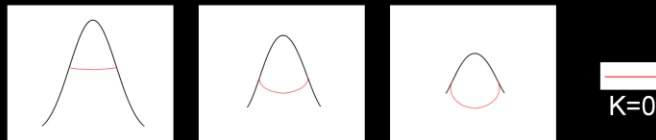
- Contours must end in a concave way

[Koenderink 1982]

- they only occur where  $K < 0$



- However, the concave ending might be hard to see



a Gaussian bump viewed from the side towards the top

A related result is that since ending contours only occur where the Gaussian curvature is negative, the contours must end in a concave way, approaching their end with negative apparent curvature.

\*\*\*\*

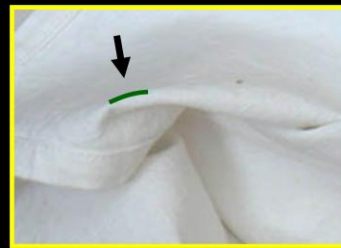
But Koenderink and van Doorn also noticed that artists tend to draw lines that are missing these concave endings.

It turns out this concave ending can be difficult to discern, as is the case for this Gaussian bump.

DEMO

# Ending contours in images

Difficult to localize



Contours are typically easy to detect in real images, at least when the lighting is right.

And there are many studies that demonstrate how people use them for visual inference.

However, in many cases, it's not easy to determine where a contour ends.

Here's an example photograph of napkin.

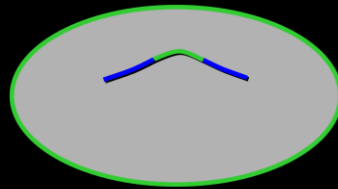
Even if we zoom in, it's still not clear whether the surface occludes itself or whether it's simply heavily foreshortened.

Observations like this make sense of line types that extend ending contours: suggestive contours and apparent ridges.

# Information in suggestive contours

Suggestive contours line up  
with ending contours

[DeCarlo 2003]



Suggestive contours are another type of line to draw, and whether they are in fact detected and represented by our perceptual processes is still an open question.

They do seem to produce convincing renderings of shape in many cases.

The fact that suggestive contours smoothly line up with contours in the image is encouraging.

\*\*\*\*

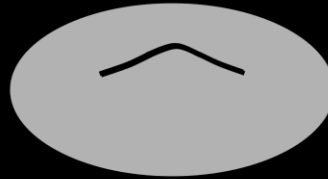
In fact, if the lines aren't color coded, it's difficult to tell where one starts and the other ends.



# Information in suggestive contours

Suggestive contours line up  
with ending contours

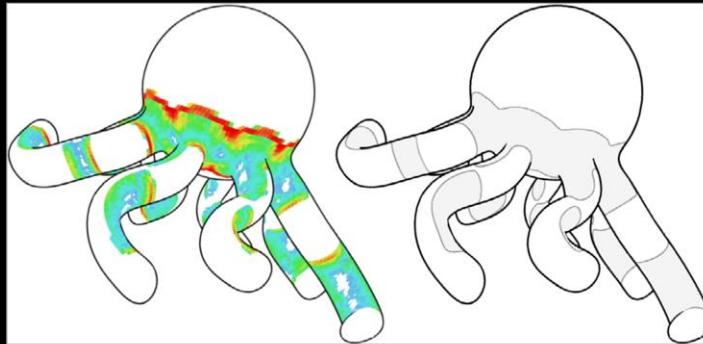
[DeCarlo 2003]



This makes it hard to tell where contour ends

# Information in suggestive contours

Can only appear in hyperbolic regions ( $K < 0$ )



Suggestive contour  
density over all views

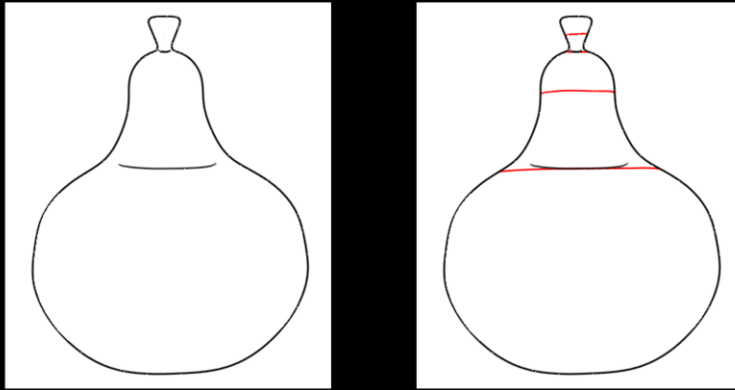
Regions where  
 $K < 0$

We can say something about what information they provide.

Recall from earlier how suggestive contours  
can only appear  
where the Gaussian curvature is negative.

# Information in suggestive contours

Away from the contour, they approach  
parabolic lines ( $K = 0$ ) [DeCarlo 2004]



In many cases, the suggestive contours  
approach the parabolic lines  
away from the contour.

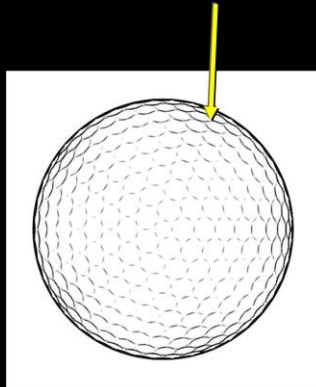
On this pear, we see how the suggestive contour  
skims along the parabolic line.

DEMO

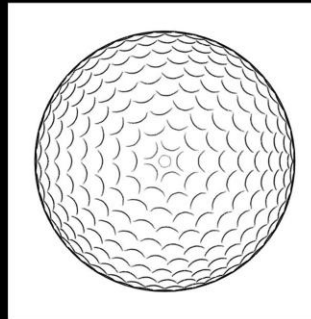
We hope to be able to say more about this in the future.

# Information in apparent ridges

Near the contour, approach suggestive contours



Apparent ridges



Suggestive contours

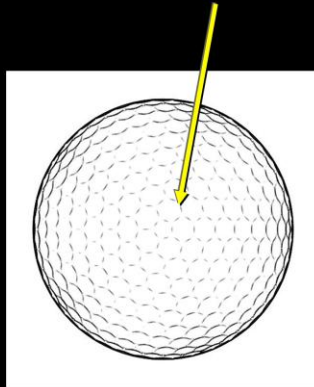
We can make similar statements about apparent ridges.

Near the contour, apparent ridges behave like suggestive contours.

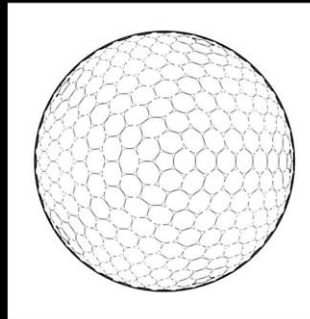
They extend ending contours.

# Information in apparent ridges

Where front-facing, approach ridges and valleys



Apparent ridges



Ridges and valleys

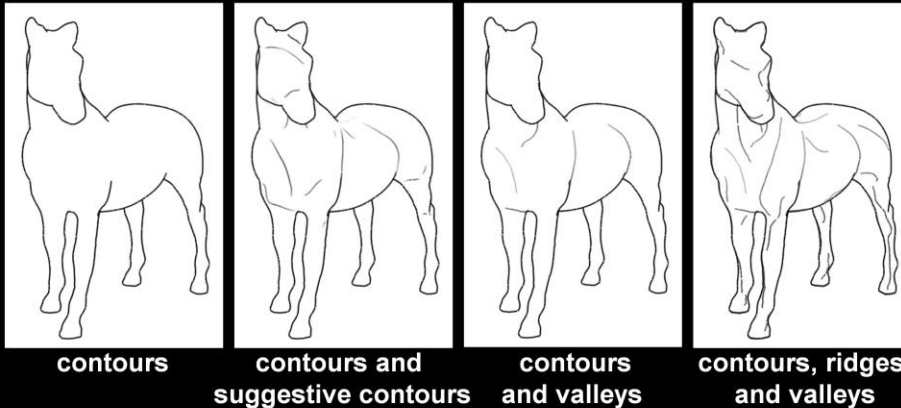
As the surface faces more towards the viewer,  
the location of apparent ridges  
approaches ordinary ridges and valleys.

And of course, in both of these cases,  
apparent ridges are surface locations  
where the normal vector is changing maximally.

# Interior shape features

What lines to draw?

– still not resolved, really



We can compare renderings with ridges and valleys to renderings with suggestive contours.

On the horse from this viewpoint, the rendering with just valleys is actually quite convincing.

As noted earlier, many of the ridges appear as surface markings here.

For the valley rendering, some features are missing, but the more salient features on the side of the horse are depicted.

Note the slight differences between the lines from suggestive contours, and from valleys.

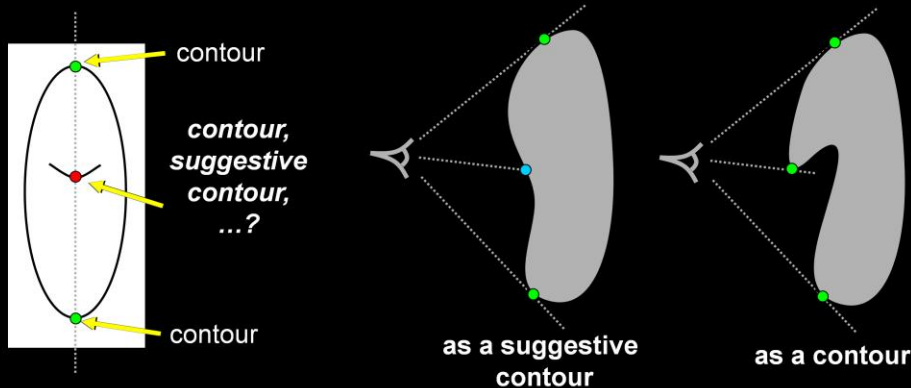
The shapes they convey appear to be a little different.

Clearly there is a lot of interesting work to do here.

This concludes our discussion of what information particular lines provide.

# Line labeling ambiguity

Different assignments of line labels correspond to different surfaces



Of course, this information can only be used if we know the TYPES of the lines when we're given a drawing.

Earlier we discussed algorithms for line drawing interpretation; approaches like this are reasonable to consider for this purpose.

But even if we do use these algorithms, there are often several different labelings that are consistent.

Given the line drawing on the left which depicts an elliptical shape with a bump, we can successfully label the green points as contours.

The red point, however, can be either a contour or suggestive contour.

Two possible shapes that match these labelings are shown on the right.

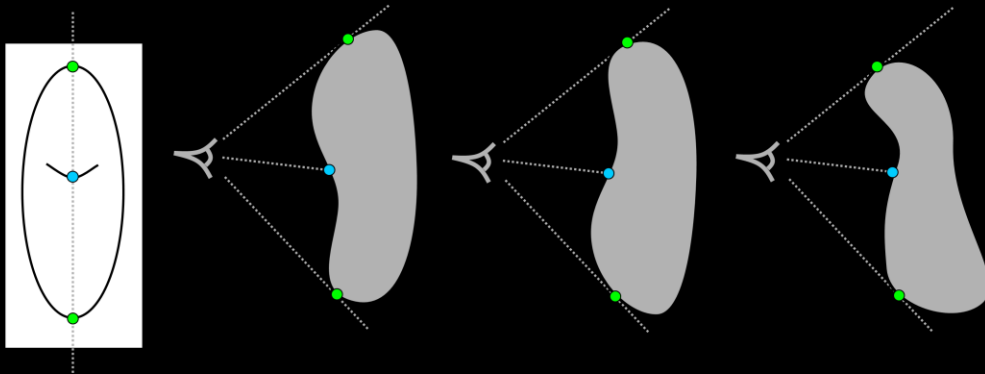
Presumably this problem cannot be solved in general.

There will always be ambiguity.

It's possible that when artists make line drawings, they're careful to shape the remaining ambiguity so it won't be a distraction.

# Projective ambiguity

Even given a line labeling, an infinity of shapes correspond to that drawing



And even with a line labeling,  
there is the ambiguity of projection.

These three interpretations have the same line labeling,  
but different geometries.

At first, this seems hopeless.

Yet sketching interfaces like Igarashi's Teddy  
seem to be quite successful by using inflation.

How can this be?

Well, there are reasonable constraints on smoothness  
that we can expect of the underlying shape.

We also presume  
that the artist has drawn all of the important lines,  
so that no extra wiggles remain.

These issues are the source of one crucial challenge  
for sketch-based shape modeling.



# Bas-relief ambiguity

The projective ambiguity that preserves planarity

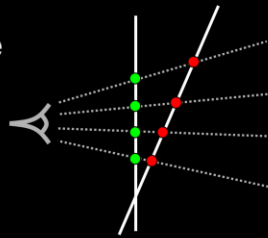
- ambiguity in Lambertian shaded imagery

[Belhumeur 1999]

- preserves contours, shadow boundaries, (relative) signs of curvature

- has perceptual significance

[Koenderink 2001]



We can be more specific with regard to this ambiguity.

For real images, there are well defined ambiguities for particular types of imagery.

One notable example is the ambiguity that remains when viewing a shape under Lambertian illumination.

There is a group of shape distortions that can be applied to a shape, that with an corresponding transformation of the lighting positions, approximately produce the same image.

This is the three-dimensional projective mapping known as the generalized bas-relief transformation.

As shown here, it moves points along visual rays and preserves planes.

It also preserves contours, boundaries of shadows, and the relative signs of curvature on the shape.

Perhaps most interestingly is that when you ask people to describe the shapes they see in shaded imagery, they answer consistently modulo this ambiguity transformation.

# Evaluation of line drawings

When is an automatic line drawing effective?

1. compare it to drawings by skilled artists
2. psychophysical measurement
  - perceived shape should be consistent with the original (modulo ambiguity)
  - bas-relief ambiguity may be appropriate  
[Koenderink et al. 1996, Belhumeur et al. 1997]
  - such methods may yield global inconsistencies  
[Li and Pizlo 2006]

So how can we be sure that a line drawing we make is perceived accurately?

As you saw earlier, one possible path is to compare that line drawing to those made by skilled artists.

Another way, based in psychophysics, is to simply ask the viewer questions about the shape they see. If this is done right, you can reconstruct their percept and compare it to the original shape, given the appropriate ambiguity transformation.

Koenderink and colleagues already performed a study like this on a single line drawing.

Their results suggest that the bas-relief ambiguity might be the appropriate one to consider here.

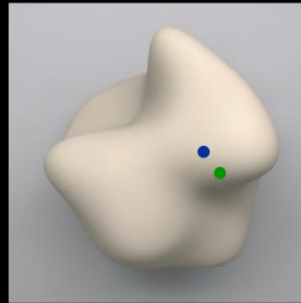
However, this ambiguity may only be resolved locally, where different parts of the shape are locally consistent, but not necessarily in a global sense.

# Psychophysical studies

## Measurement of perceived shape

[Koenderink 2001]

- depth probing
  - asks the viewer which point is closer (green or blue)



So what kinds of questions can you ask viewers?

In psychophysics, the answer is:  
very simple ones, and lots of them.

Koenderink describes a set of psychophysical methods  
for obtaining information  
about what shape a viewer perceives.

The first they describe is called **RELATIVE DEPTH PROBING**.

The viewer is shown a display like this one,  
and is simply asked which point appears to be closer.

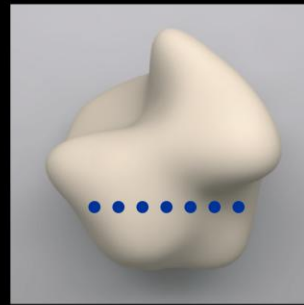
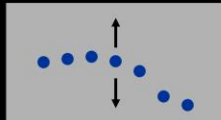
They are asked this question for many pairs of points.

# Psychophysical studies

## Measurement of perceived shape

[Koenderink 2001]

- **depth profile adjustment**
  - viewer adjusts points until they match the profile of a particular cross-section



Another method is known as **DEPTH PROFILE ADJUSTMENT**.

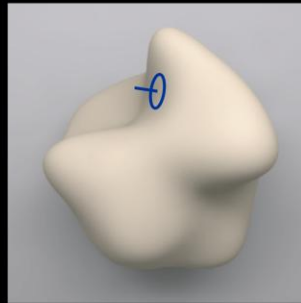
Here, the viewer adjusts points to match the profile of a particular marked cross-section on the display.

# Psychophysical studies

## Measurement of perceived shape

[Koenderink 2001]

- gauge figure adjustment
  - viewer adjusts a disk until it appears to sit on the tangent plane of the surface



Their third method is known as GAUGE FIGURE ADJUSTMENT.

Here, the viewer uses a trackball to adjust a small figure that resembles a thumbtack, so that it looks like its sitting on the surface.

All of these methods are successful.

But gauge figure adjustment seems to give the best information given a fixed number of questions.

# Summary

- Specific information is conveyed by each type of line
- Psychophysical evidence suggests humans use this information
- How exactly humans interpret line drawings is still unknown

So in summary,

Each type of line in a line drawing conveys specific information about shape.

There is a fair amount of evidence that people use this information.

But how exactly people use this information is still unknown.

We're very encouraged by how a combination of computer graphics and psychophysics can lead to answers to these questions.